USING THE DIFFERENTIAL EVOLUTION ALGORITHM FOR PROCESSING STAR CAMERA MEASUREMENTS

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Abstract: The development of star cameras started in the 1970s. Their purpose was the determination of astronomic position. Using photogrammetric method, the fieldwork could be carried out quickly. The accuracy was reduced compared to the traditional methods, yet the overall time needed for measurement and processing decreased drastically. In the 2000s, some of these instruments were fitted with CCD sensors. Automatized data processing methods were developed, which provided superior speed and accuracy. Recently, the development of a simplified star camera system has been started at the HAS-BUTE PGG. In this paper some key steps of the processing of the measurements are outlined. Many of these steps can be regarded as optimization problems. For this purpose, the Differential Evolution was chosen as a fitting algorithm.

Keywords: Star camera, Evolutionary algorithm, Differential evolution, Least squares, Adjustment

1. Introduction

1.1. Geodetic and astronomic position

Given a reference surface placed in a Cartesian coordinate system, the location of a point can be given by three Cartesian coordinates. Equivalently, if the location of the point is restricted to be on the reference surface, its position can be expressed by the direction of the surface normal at that point. This direction is usually given by two polar coordinates, which are called latitude and longitude (see Fig. 1).
Fig. 1. Reference surface, point ‘P’ on the surface, and its position given by Cartesian or polar coordinates

The reference surface can be chosen to be the so-called geoid. The geoid is an equipotential surface of the gravity field of the Earth, which is placed at the mean sea level. Thus at oceanic areas it is equivalent to the mean sea level itself, and under the continental areas it is extended. In this case, the direction of the surface normal in a point is equivalent to the local vertical direction at that point. In this case, the position is called astronomic position, and the angles are called astronomic latitude $\Phi$ and astronomic longitude $\Lambda$. Astronomic positions can be determined by measurements on stars and other celestial objects.

As the shape of the geoid is too complicated to deal with in everyday life, a simpler reference surface must also be chosen to be used in geodesy and surveying. This surface is usually an ellipsoid. The flatting, size and placement of the ellipsoid is chosen to suit some local or global need (i.e. to optimally approximate the shape of the geoid in a country, on a continent, or globally). In this case, the position is called geodetic position, and the angles are called geodetic latitude $\phi$ and geodetic longitude $\lambda$. Geodetic positions can be determined by e.g. navigational satellite systems.

Astronomic position data can be used e.g. for determining the optimal parameters of a reference ellipsoid, determining the precise parameters of the Earth’s gravity field, etc. Geodetic positions play a significant role in everyday life: they are used for navigation, mapping, construction controlling, etc.

The difference between the two kinds of position is called the deflection of the vertical $\Theta$. The deflection can be decomposed into two components, one in the direction of the latitude $\xi$, and the other in the direction of the longitude $\eta$. In Hungary, its usual order of magnitude is some arc-seconds.

1.2. Star cameras

In the 1970s, development of star cameras (or zenith cameras) had started in Germany, Italy and Austria. These cameras could be used to determine astronomic
positions in a simple and (compared to other methods at that time) quick way: photos of the starry sky were taken in zenith direction, thus the direction of the zenith (which, as stated before, is the same as the astronomic position) could be calculated by simple transformation, based on the known direction of the stars. The accuracy of the instruments was not as good as the accuracy of traditional methods, based on the measurement of horizontal and vertical angles of stars, but the time needed for measurements and processing decreased drastically (some hours vs. some weeks).

Until the 1990s these devices were widely used in Europe. Since 2000 two of them have been renovated in Hannover and in Zürich, using high sensitivity CCD sensors [1]. Fully automatized measurement and data processing techniques were developed. The increased accuracy of the input data (e.g. star catalogues, timing of the measurements), the increased accuracy of the photos and their evaluation (high resolution of the sensors and moment-based light source evaluation), and the quick and automatized processing method together mean superior speed and precision. Today these instruments are the state-of-the-art solution for astronomic positioning.

1.3. Simplified star camera system

Recently the development of a simplified star camera system (see Fig. 2) has been started at the HAS-BUTE Research Group for Physical Geodesy and Geodynamics (PGG) [2]. The main goal of the project is to create a system which is built from simple, commercially available parts, and to test whether a processing software can be developed, which is capable to compensate the less advanced hardware solutions.

Fig. 2. Simplified star camera system

The base instrument is a traditional surveying theodolite (Zeiss THEO010B). On the top of it, a zenith-facing digital camera (Canon EOS350D) is fitted. As stated before, under ideal circumstances, the astronomical position can be calculated by transforming the zenith direction into the coordinate system of the stars. In reality, the optical axis of the camera does not coincide with the vertical axis of the base instrument (the small angle enclosed by the two is the angle of misalignment on Fig. 2, hereafter marked
as $\Theta$). As the instrument can be rotated around its vertical axis, the direction of the optical axis of the camera sweeps a cone. The axis of revolution of this cone is the vertical direction. The small tilt of the axis of rotation of the theodolite w.r.t. the true local vertical direction can be measured and taken into account. Thus, the steps of the processing are the following:

1. Identifying light sources on the digital pictures of the starry sky. That is, the pixels of the images, which possibly correspond to stars, have to be identified. Sub-pixel accuracy is desired;
2. Matching the identified light sources and the corresponding stars. That is, using the available star catalogue data, the corresponding stars on the images and in the catalogue have to be paired;
3. Determining the direction of the camera's optical axis, for every image;
4. Determining the axis of revolution of the cone swept by the optical axis.

Step 2 is a general fitting problem. Steps 3 and step 4 can be treated as least squares parameter estimation problems. The solution of these problems relies on the Differential Evolution (DE) algorithm, a global non-linear optimization method related to the family of evolutionary algorithms, developed in the 1990s [3]. In the following, these steps, their mathematical background, and the application of DE for them are discussed.

### 2. Algorithms of processing

#### 2.1. Detecting light sources

First of all star candidates on the images have to be detected by detecting coordinates of local pixel value maxima. The green channel of the RGB images is used for two reasons:
- it has the least noise among the channels;
- it has the best native resolution (given the ‘GRBG’ Bayer arrangement of the image sensor).

After detecting all the local maxima, an aggressive thresholding is applied on the image (approx. 99.9% of the pixel values are zeroed out), to eliminate noise. The last step is to calculate the weighted mean of the remaining pixel values surrounding each local maximum, which gives a sub-pixel estimation of the image coordinates of the light source (see Fig. 3).

#### 2.2. Matching light sources and stars

The next step is to match the identified light sources and the corresponding stars. As mentioned before, that means, that using the available star catalogue data, the corresponding stars on the images and in the catalogue have to be paired. The difficulty of this step is that the content of the pictures and the star catalogue does not completely overlap: some stars in the catalogue are missing from the pictures (e.g. because of clouds or other obstacles), whereas some light sources on the pictures are not in the...
catalogue (e.g. they are below a given visual magnitude limit, or they are not real stars but other shiny objects, sensor errors, etc).

Fig. 3. Part of an image surrounding a star candidate, before (left) and after (right) thresholding, ‘X’ marks the weighted mean position

First of all, a preliminary image of the sky is calculated, by the principle of central projection (see Fig. 4). This image is ‘preliminary’, because it is based on a priori knowledge of the following parameters:

- position of the camera’s projection center at the time of taking the photo;
- direction of the camera’s optical axis at the time of taking the photo;
- internal properties of the camera.

Fig. 4. The principle of central projection
Since the ratio of the radius of the Earth to the distance of the stars is very small, the position of the camera’s projection center can be taken as if it was in the center of mass of the Earth, causing negligible error, that is, the ‘daily parallax’ is neglected.

An approximate value of the direction of the optical axis at the time of taking the photo can be determined by GPS measurements during the fieldwork. Needing only an approximate value, the inaccuracy of the GPS equipment (navigation-grade GPS receiver has been used, giving an accuracy of some meters), the deflection of the vertical, and the tilt of the camera axis with respect to the axis of revolution are neglected.

The internal properties of the camera are the coordinates of the projection center with respect to a coordinate system fixed to the image sensor. One of the three coordinates is the camera constant (the distance of the projection center from the sensor, in a direction perpendicular to the sensor, which is equivalent to the focal length of the camera if the lens is focused to the infinity), which is known from earlier experiments. The other two coordinates express the shift of the projection center from the center of the image sensor, in the direction parallel to the sensor itself. The values of these two coordinates can safely be taken as zero.

Position of the stars has been given by an electronic star catalogue. Here the program ‘XEphem’ has been used. It is a scientific-grade free/open source interactive astronomical ephemeris software package for UNIX-like systems [4]. The data used by the program is a subset of the SKY2000 Master Catalog, Version 4 (containing only stars with magnitude 6.5 and higher). As the position of the stars is given in a barycentric coordinate system (International Celestial Reference System, ICRS) for the epoch J2000.0, some calculations are needed to get the directions of the stars in an Earth-Centered-Earth-Fixed (ECEF) system for the epoch of taking the images. The proper motion of the stars, the yearly parallax, yearly aberration, nutation and precession are calculated by the software. Other corrections are also calculated based on the ‘IERS Bulletin B’ data made available by the International Earth Rotation and Reference Systems Service (IERS); these are the differences caused by the rotation of the Earth, and by the small motion of the rotation pole [5].

Knowing these quantities, the preliminary image coordinates of the catalogue stars can be calculated by the principle of central projection. Then, using a simple 3-parameter transformation

\[
\begin{bmatrix}
\xi_T \\
\eta_T
\end{bmatrix} = R(\alpha) \cdot \begin{bmatrix}
\xi \\
\eta
\end{bmatrix} + \begin{bmatrix}
\xi_0 \\
\eta_0
\end{bmatrix},
\]

the real image coordinates can be transformed so that a fair match between the real image data and the preliminary coordinates is found (see Fig. 5). Here \(\xi\) and \(\eta\) are the original image coordinates, \(\xi_T\) and \(\eta_T\) are the transformed coordinates, \(R(\alpha)\) is the 2x2 rotation matrix, \(\alpha\) is the rotation angle, and \(\xi_0\) and \(\eta_0\) are the translation parameters.

The degree of match can be expressed many ways. In this case the following method has been chosen: given a transformation parameter set \((\alpha, \xi_0, \eta_0)\), the transformed image coordinates \((\xi_T, \eta_T)\) of the light sources are calculated. Then the image distances of every transformed light source from every preliminary star position (distance matrix)
are calculated. The sum of the squared image distances of the five best matching light sources and stars gives us the quantitative expression of the match. This is the cost function (or objective function) to be minimized using the DE algorithm.

![Fig. 5. Light sources on the image, preliminary image, and matching the two after finding the optimal transformation parameters](image)

Because the star catalogue and the image data does not completely overlap for reasons detailed at the beginning of this section, not all the light sources of the image, or all the stars from the catalogue should be taken into account. In the other hand, taking into account too few light sources or stars may result in false matches. Five light sources is a fair mean, which has proven to have good convergence properties, while giving a stable solution.

After calculating the optimal transformation parameters, the final pairing of the light sources and stars can be done. The distance matrix is calculated again, and the matching pairs are selected by simple thresholding.

2.3. Determining the direction of the optical axis

After the light sources and stars have been matched for all the images, the direction of the camera’s optical axis for every image can be determined. The principle of central projection is utilized again, but this time the precise direction of the optical axis and the precise value of the camera constant is sought for. The direction of the camera is expressed with three Euler rotation angles, $\Phi$, $\Omega$ and $\kappa$, with respect to the $Z$, $X$ and $Z$ axis of the ECEF coordinate system. Two of the three rotation parameters has direct connections with the astronomic direction of the optical axis. The third one represents the camera’s rotation about its own optical axis, thus it is related to the azimuth, in which the image has been taken.

The position of the projection center and the other two internal camera properties remain fixed at their preliminary values, because they cause negligible error in the final astronomic position determined.
This time not only the five best matching stars are used, but all the identified and paired stars from the image. During the calculation, the ECEF star coordinates are effectively transformed into a Projection-Center-Fixed (PCF) coordinate system (cf. Fig. 4) of the camera by three rotations

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = R_Z(\Phi) \cdot R_X(\Omega) \cdot R_Z(\Omega) \cdot \begin{bmatrix} X \\
Y \\
Z
\end{bmatrix}.
\]  

(2)

Here \(X, Y\) and \(Z\) are the ECEF coordinates of the star (the three Cartesian components of the unit vector pointing into the direction of the star); \(R_X()\) and \(R_Z()\) are 3×3 rotation matrices around the index-indicated axis; \(\Phi, \Omega\), and \(\Omega\) are the before mentioned Euler angles; and \(u, v\), and \(w\) are the Cartesian components of the unit vector transformed into the PCF coordinate system.

After this transformation, the image coordinates of the stars can be calculated by simple proportions (cf. Fig. 4)

\[
\xi_{\text{calc}} = \frac{-u}{w} \cdot c, \quad \eta_{\text{calc}} = \frac{-v}{w} \cdot c,
\]

(3)

where \(\xi_{\text{calc}}\) and \(\eta_{\text{calc}}\) are the calculated image coordinates of the stars; \(u, v\), and \(w\) are the star coordinates calculated before in the PCF system, \(c\) is the before mentioned camera constant, and the two remaining internal camera parameters are taken to be zero.

Now that the calculated image coordinates of the stars are known, the pair-wise image distances of the calculated image positions and the measured image positions \(\xi_{\text{meas}}, \eta_{\text{meas}}\) can be determined. The sum of the squared distances is the cost function to be minimized using the DE algorithm (see Fig. 6), looking for the optimal rotation angles \(\Phi, \Omega, \Phi\), and camera constant \(c\)

\[
\sum (\xi_{\text{meas},i} - \xi_{\text{calc},i})^2 + (\eta_{\text{meas},i} - \eta_{\text{calc},i})^2_{\Phi, \Omega, K, c} \rightarrow \min.
\]

(4)

After these parameters are found, the direction of the optical axis can be determined in the ECEF system by transforming back the sensor normal direction

\[
\begin{bmatrix} X \\
Y \\
Z
\end{bmatrix} = R_Z(-K) \cdot R_X(-\Omega) \cdot R_Z(-\Phi) \begin{bmatrix} u \\
v \\
w
\end{bmatrix} 
\]

\[
\mid u = 0, v = 0, w = 1
\]

(5)

2.4. Determining the astronomic position

As stated before, the astronomic position of the instrument is equivalent to the direction of the vertical axis of the theodolite, which in turn is equivalent to the axis of
revolution of the cone swept by the optical axis of the camera while rotated into different positions. Thus the next step is to determine the axis of revolution.

Assuming that the direction of the optical axis of the camera is known at every position (expressed by the coordinates \(X_i, Y_i\) and \(Z_i\) of the vectors pointing in the directions of the axis), the angle of misalignment \(\Theta_i\) enclosed by the optical axis and the axis of revolution (expressed by the coordinates \(X_R, Y_R, Z_R\)) at each position can be calculated as the following

\[ \Theta_i = \arccos\left( \frac{X_i \cdot X_R + Y_i \cdot Y_R + Z_i \cdot Z_R}{\sqrt{X_i^2 + Y_i^2 + Z_i^2} \cdot \sqrt{X_R^2 + Y_R^2 + Z_R^2}} \right) \]
\[ \theta_i = \arccos \left( \frac{X_{R} \cdot X_i + Y_{R} \cdot Y_i + Z_{R} \cdot Z_i}{\sqrt{X_{R}^2 + Y_{R}^2 + Z_{R}^2}} \right) \]. \quad (6) \]

The goal is to determine the direction of the axis of revolution that minimizes the variance of the enclosed angles (where \(N_\Theta\) is the number of the camera positions)

\[ \frac{\sum (\theta - \theta_i)^2}{N_\Theta - 1} \rightarrow \min. \quad (7) \]

This optimization problem has also been solved by the DE algorithm (see Fig. 7).

Fig. 7. Optical axis directions (crosses), section of the fitted cone (continuous line) and the direction of its axis of revolution (circle) after 1, 10, and 40 iterations

2.5. Development environment

The algorithms described in this chapter have been implemented in the Octave [6] computing environment (which is a free/open source computer algebra system, highly compatible with the popular MATLAB environment). The tool used for performing auxiliary astronomical computations was the AA 5.5 program [7] (also free/open source). It is worth to mention, that although it has not been done, the DE algorithm is easy to implement in a parallel programming environment [8].

3. Summary

In the preceding chapter it has been shown, how the DE algorithm can be used to process measurements made with the simplified star camera system. The described algorithm has been implemented using the Octave computer algebra system and the AA tool. The developed processing software is almost completely automatized.

So far, the results of two test measurements (24/10/2008 Budapest, 04/06/2009 Balatonkenese) have been processed with the developed system. The precision varies between the two tests. Investigations so far have suggested that the reason for the
varying precision lies in the technical details of carrying out the measurements, not in the processing software.

In the future, the most important task is to carry out more test measurements. If possible, these measurements should be repeated tests on the same site, under various meteorological conditions. Thus, after determining the source of some systematic errors, the precision of the system and the influence of meteorology could also be estimated.

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