GEODETIC APPLICATIONS OF TORSION BALANCE MEASUREMENTS IN HUNGARY

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SUMMARY

First the operating principle of torsion balance is outlined, then a short history of measurements in Hungary is discussed. Finally some practical geodetic applications are presented.

INTRODUCTION

Sooner or later very precise and simple gradiometers will be available for geoscientists. By constructing suitable instruments, gravity gradients will be measurable with high accuracy simply and quickly. In this regard it may be important to investigate the geodetic applications of earlier torsion balance measurements.

Knowledge of gravity gradients is very important for geodesy. Gravity gradients are the elements of gravity gradient tensor (Eötvös tensor):

$$\mathbf{E} = \operatorname{grad}\mathbf{g} = \operatorname{grad}(\operatorname{grad}W) = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix} .$$

$\mathbf{E}$ is a symmetrical tensor, because $W_{xy} = W_{yx}$, $W_{xz} = W_{zx}$, $W_{yz} = W_{zy}$. The third line of tensor $\mathbf{E}$ represents the gravity gradients: W_{zx} and W_{zy} are the two components of the *horizontal gradient* and W_{zz} is the *vertical gradient*. Horizontal gradients of gravity W_{zs} can be illustrated as vectors having the length of $W_{zs} = \sqrt{W_{zx}^2 + W_{zy}^2}$ and azimuth $\alpha = \arctan(W_{zy} / W_{zx})$ (VÖLGYESI, 1999). Furthermore $\mathbf{E}$ contains the $W_\Delta = W_{yy} - W_{xx}$ and W_{xy} *curvature data* - using the terms of Eötvös (SELÉNYI, 1953). Curvature of level surfaces can be illustrated as line segments having the length of $R = \sqrt{W_\Delta^2 + 4W_{xy}^2}$ and the azimuth of maximal curvature $\alpha = 1/2 \arctan(-2W_{xy} / W_\Delta)$ (EGYED, 1955). It is important to know that W_Δ and $2W_{xy}$ characterize the curvature of potential surfaces

(how the shape of potential surfaces differ from the shape of a sphere), and W_{zx} , W_{zy} characterize how potential surfaces are not parallel to each other.

Torsion balance can measure the components of *horizontal gradient* W_{zx} , W_{zy} and the *curvature data* W_{Δ} , $2W_{xy}$ - but unfortunately it can't measure the *vertical gradient* W_{zz} .

1. PRINCIPLE OF TORSION BALANCE

The *Eötvös torsion balance* consists of a horizontal beam having the length $2l$ with masses m on each ends suspended at a torsion wire. One of the two masses is on the one end of the horizontal beam, and the other mass is suspended at a distance h from the other end of the beam - as it can be seen on *Fig. 1*. Horizontal component of gravity acting on the two masses causes a torque, and the horizontal beam is rotated until an equilibrium position with the restoring torque of the suspending torsion wire (having the torsion constant τ) is reached. In the equilibrium condition of torques the scale reading is n , while the scale reading of the torsion-free zero position of the beam would be n_0.

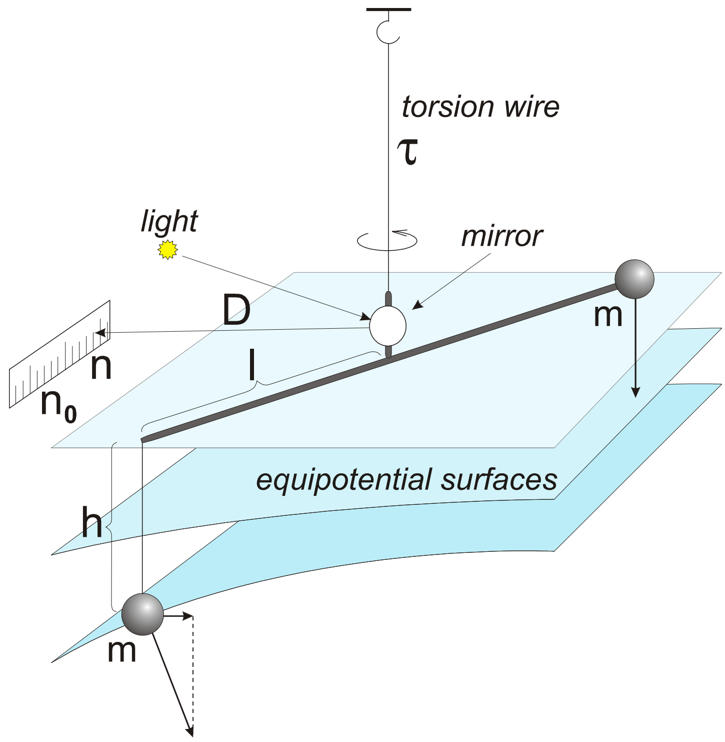

Fig. 1 Principle of torsion balance

The base equation of Eötvös torsion balance is:

$$n - n_0 = \frac{DK}{\tau}\left(W_{\Delta} \sin 2\alpha + 2W_{xy} \cos 2\alpha\right) + \frac{2lDhm}{\tau}\left(W_{zy} \cos\alpha - W_{zx} \sin\alpha\right) \qquad (1)$$

where K is the moment of inertia, and α is the azimuth of the beam. The earlier type of instrument is the *Cavendish* torsion balance, in which the two masses are on the same height on the two ends of the beam. This type of instrument is unable to measure the components of *horizontal gradient* W_{zx} and W_{zy}, because $h = 0$ in Eq. (1).

Based on Eq. (1) there are five unknowns $(n_0, W_\Delta, 2W_{xy}, W_{zx}, W_{zy})$ at each measuring site, so the readings should be made in five different azimuths. Usually two beam systems are mounted in one instrument at antiparallel position to each other, so there is a new unknown torsion-free zero position $n_0^{'}$ for the new beam system. Due to the additional unknown, six measurements in three different azimuths (e.g. 0^0, 120^0, 240^0) are sufficient, but it is necessary to repeat the measurements in order to increase the accuracy.

Different types of torsion balances were produced mainly by the Hungarian company *Süss* and *ELGI*, and *Askania* in Berlin. Before the year 1925 only the non series instruments were made, but after this time more than 300 torsion balance instruments were made in series in Hungary. The main parameters of the most important Hungarian torsion balance instruments were summarized in *Tab. 1* (SZABÓ, 1999).

Tab. 1 Main parameters of the most important type of instruments
m - mass [g], *l* - half of the length of the beam [cm], *h* - distance between the beam and the suspended mass [cm], *L* - length of the suspending torsion wire [cm], *d* - diameter of the torsion wire [mm], τ - torsion constant of torsion wire [cgs], *D* - distance between the mirror and the scale [cm], *K* - moment of inertia of the beam [cgs], *T* - decay time [min].

Type	*year*	*m*	*l*	*h*	*L*	*d*	τ	*D*	*K*	*T*
Auterbal	1925	15	7	22	20	0.017	0.03	32	1700	40
Pekar 2b	1930	12	10	32	30	0.020	0.07	45	2450	45
E-54	1954	9	10	30	20	0.019	0.06	31	1900	40
E-60	1960	9	10	30	20	0.022	0.20	31	1900	20

The sensitivity of torsion balance can be increased by extending the oscillation time, which can be achieved by increasing K and decreasing τ. Depending on the aperiodic air damping, a large oscillation time implies long time period for the system to come to the rest position where a reading is made (TORGE, 1989). The time required for the survey at one station was 3 - 8 hours, depending on the length of settling time T.

2. TORSION BALANCE MEASUREMENTS IN HUNGARY

The first measurements with torsion balance were made by Lorand Eötvös himself at the foot of *Gellért Hill* in Budapest in the year 1889, than at *Sághegy* near Celldömölk in 1891. The first measurements extending over a bigger area were made on winter time on the ice surface of the lake *Balaton* in 1901 and 1903, and the first successful geophysical exploration was made at Morvamezö near *Egbell* in 1916. Between 1901 and 1967 more than 60000 torsion balance measurements were made in Hungary by the companies of MAORT, ELGI and OKGT, but now approximately 5000 points are found over the Hungarian border from this 60000 stations. Points mainly cover the Great Hungarian Plain, and the hilly Transdanubian area with moderate topography, because due to the strong influence of topographic masses torsion balance measurements could only be performed in flat terrain or moderate hills. For the terrain reduction a leveling was performed at each point, to a range of 100 m around the station, and the topographic effects were computed for each point.

Based on the figures found in literature the accuracy of torsion balance measurements of gravity gradients and curvature parameters are approximately ±1 E (1 E = 1 Eötvös Unit = $10^{-9}\,s^{-2}$). There is a good possibility to check this value. In 1946 calibration measurements were completed at the point Üllö_1 ($\varphi = 47^o 22' 41''$, $\lambda = 19^o 20' 42''$ in WGS-84 system) - near to Budapest - and the field books contain interesting and important data concerning to the accuracy of torsion balance measurements. At that time three instruments (*No.36690*, *No.37265* and *No.37286*) were set up on the three points of a triangle (*Üllö_1/a*, *Üllö_1/b* and *Üllö_1/c*) at a distance of approximately 15 - 20 m to each other on a flat ground surface with small sand-dunes. The results of calibration measurements are summarized in *Tab. 2*.

Tab. 2 Calibration measurements of different instruments

	Point	Instrument *No.36690*	Instrument *No.37265*	Instrument *No.37286*	mean
W_{zx}	Üllö_1/a	0.6	1.5	0.6	0.9 ±0.3
	Üllö_1/b	-4.5	-4.2	-2.6	-3.8 ±0.8
	Üllö_1/c	-2.4	-4.2	-2.6	-3.1 ±0.8
	mean:	-2.1 ±2.1	-2.3 ±2.7	-1.5 ±1.5	-2.0 ±2.1 / ±0.6
W_{zy}	Üllö_1/a	3.4	5.4	3.9	4.2 ±0.8
	Üllö_1/b	3.1	2.2	2.7	2.7 ±0.4
	Üllö_1/c	-2.6	-1.3	-1.4	-1.8 ±0.6
	mean:	1.3 ±2.8	2.1 ±2.7	1.7 ±2.3	1.7 ±2.6 / ±0.6
W_{Δ}	Üllö_1/a	-7.8	-7.8	-8.8	-8.1 ±0.5
	Üllö_1/b	-5.1	-6.7	-7.0	-6.3 ±0.8
	Üllö_1/c	-13.6	-17.1	-11.0	-13.9 ±2.5
	mean:	-8.8 ±3.5	-10.5 ±4.7	-8.9 ±1.7	-9.4 ±3.3 / ±1.3
$2W_{xy}$	Üllö_1/a	8.8	4.7	3.3	5.6 ±2.3
	Üllö_1/b	6.0	8.8	1.9	5.6 ±2.8
	Üllö_1/c	1.5	-1.1	3.9	1.4 ±2.0
	mean:	5.4 ±3.0	4.1 ±4.1	3.0 ±0.8	4.2 ±2.6 / ±2.4

Some important conclusions may be drawn from these data - in a good accordance with other earlier observations in Hungary:

- Measuring by different instruments at the same point there are smaller standard deviations, but measuring by the same instrument at different neighboring points within a distance to each other not more than 15 - 20 m, there are bigger standard deviations.

- Standard deviations of curvature data W_{Δ} and $2W_{xy}$ are approximately two times bigger than the standard deviations of gravity gradients W_{zx} and W_{zy}, that is the gravity gradients can be measured with higher accuracy. In circumstances of fieldwork, standard deviations of gravity gradients are not bigger than ± 1 - 2 E , while standard deviations of curvature data are maximum ± 2 - 4 E in accordance with other observations.

- Accuracy of different instruments may be a little different.

- Gravity gradients and curvature data may change their values by some Eötvös Units within a flat ground surface area having the extension not bigger than 10 - 20 m, so the elements of Eötvös tensor may have changes with very high frequency and relatively small amplitudes in a completely flat area too.

Because of these facts mentioned above, mostly before the year 1960 torsion balance measurements were usually made simultaneously by two instruments at each station.

The majority of original field books of torsion balance measurements are stored at Lorand Eötvös Geophysical Institute. From the year 1995 data of these field books were started to be saved to computer files by the financial support of the *Hungarian National Research Fund* (OTKA). At present 14235 torsion balance measurements are available for further processing in computer database. These data mainly cover the Hungarian Plain, as it can be seen on *Fig. 2*.

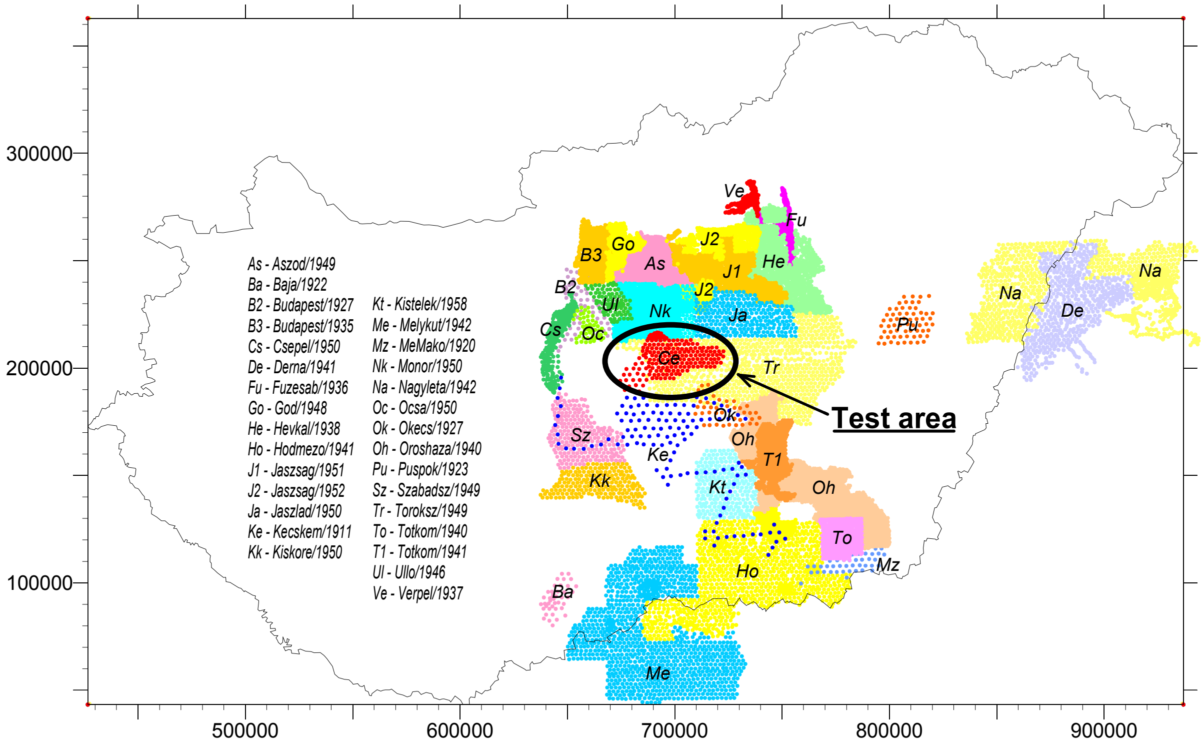

Fig. 2 Torsion balance measurements in Hungary

Since earlier torsion balance measurements were made mainly for purposes of geophysical prospecting, mostly the gravity gradients W_{zx} and W_{zy} have been processed. Up to now, the gravity curvature values W_{Δ} and $2W_{xy}$ - essential in geodesy - have been left unprocessed.

3. GEODETIC APPLICATION

Loránd Eötvös was the first to point out that interpolation of deflection of the vertical is possible from torsion balance measurements (SELÉNYI 1953). The method of Eötvös was developed in a simplified form by Renner (RENNER 1957). Further investigations were made by Badekas and Mueller (BADEKAS - MUELLER 1967), as well as Heineke (HEINEKE 1978).

A simple equation can be written for components of deflection of the vertical ξ , η between two arbitrary points i and k as well as for gravity curvature values W_Δ and $2W_{xy}$ measured by torsion balance:

$$\xi_k \sin\alpha_{ik} + \eta_k \cos\alpha_{ik} - \xi_i \sin\alpha_{ik} - \eta_i \cos\alpha_{ik} = \qquad (2)$$
$$\frac{s_{ik}}{4g}\Big[\big((W_\Delta - U_\Delta)_i + (W_\Delta - U_\Delta)_k\big)\sin 2\alpha_{ik} + \big((W_{xy} - U_{xy})_i + (W_{xy} - U_{xy})_k\big)2\cos 2\alpha_{ik}\Big]$$

where s_{ik} is the distance between points i and k , g is the average value of gravity between them, α_{ik} is the azimuth between the two points, and

$$U_\Delta = 10.26\cos^2\varphi$$
$$U_{xy} = 0$$

are normal values of gravity curvatures (VÖLGYESI, 1993). The mathematical basis of (2) is a line integration of W_Δ and $2W_{xy}$. Using (2) it is possible to interpolate deflections of the vertical for a whole measurement network if some points have known deflection values.

A software was developed for computations which can be used to determine deflections of the vertical by any method of interpolation either along chains or in networks covering arbitrary area (VÖLGYESI, 1995). Test computations were performed in an area extending over some 1200 km^2 (an ellipse shows the position of this test area on *Fig. 2*) and well measured by torsion balance, where both topographic conditions and the density of torsion balance measurements and astrogeodetic stations reflects average conditions in Hungary; and there was a possibility to check calculations because astrogeodetic and astrogravimetric data were available. In *Figs. 3* and *4* gravity gradients W_Δ and $2W_{xy}$ measured by torsion balance are visualized on the test area. The interpolation network has 203 points with unknown deflections of the vertical and geoid undulations, and there are 6 points where absolute ξ , η and N values are known on this area of investigation, referring to the *GRS80* system. (3 points were used for interpolations, and 3 points for checking of computations.)

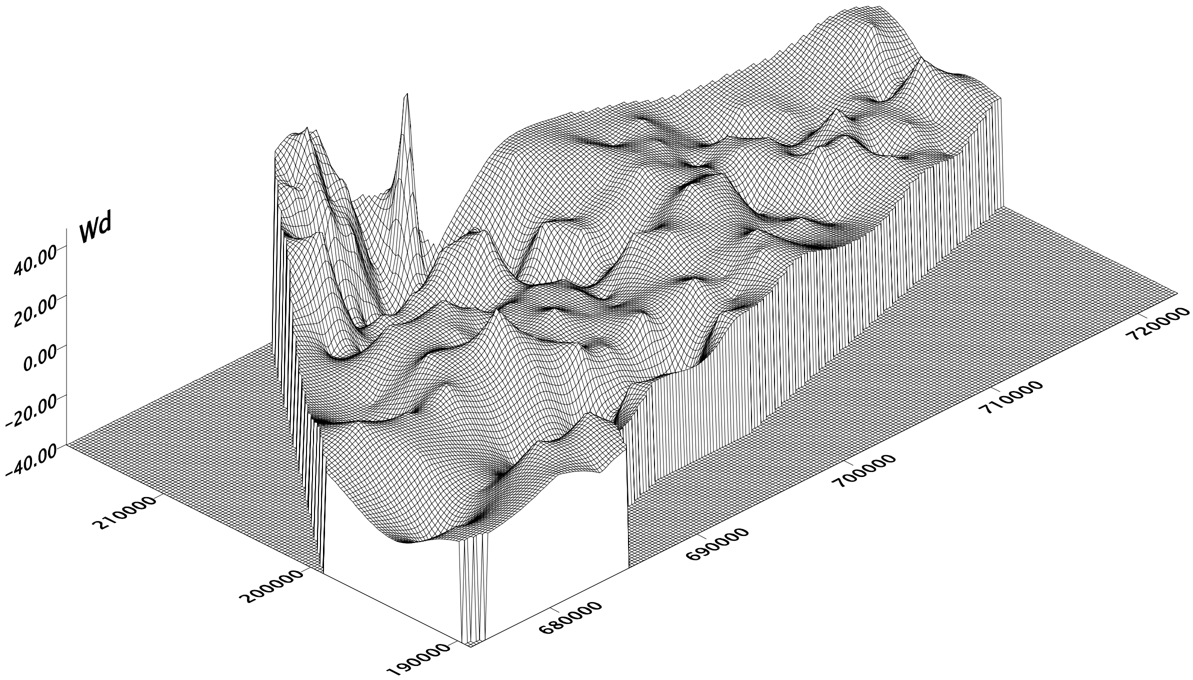

Fig. 3 *g*ravity gradients W_{Δ} on the test area

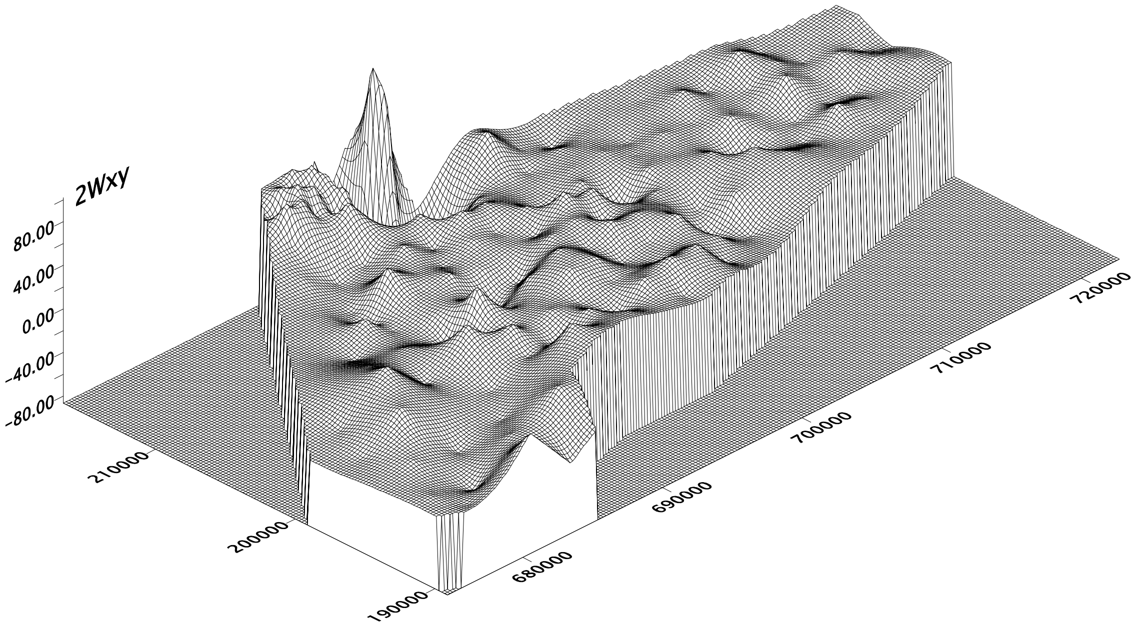

Fig. 4 *g*ravity gradients $2W_{xy}$ on the test area

Interpolated ξ and η components of deflections of the vertical that resulted from the computation visualized on isoline maps in *Figs. 5* and 6. Standard deviations $m_\xi = \pm 0.60''$ and $m_\eta = \pm 0.65''$, computed at checkpoints confirm the fact that even for large continuous territories ξ, η values of acceptable accuracy can be computed from torsion balance measurements (VÖLGYESI, 1995, 1998).

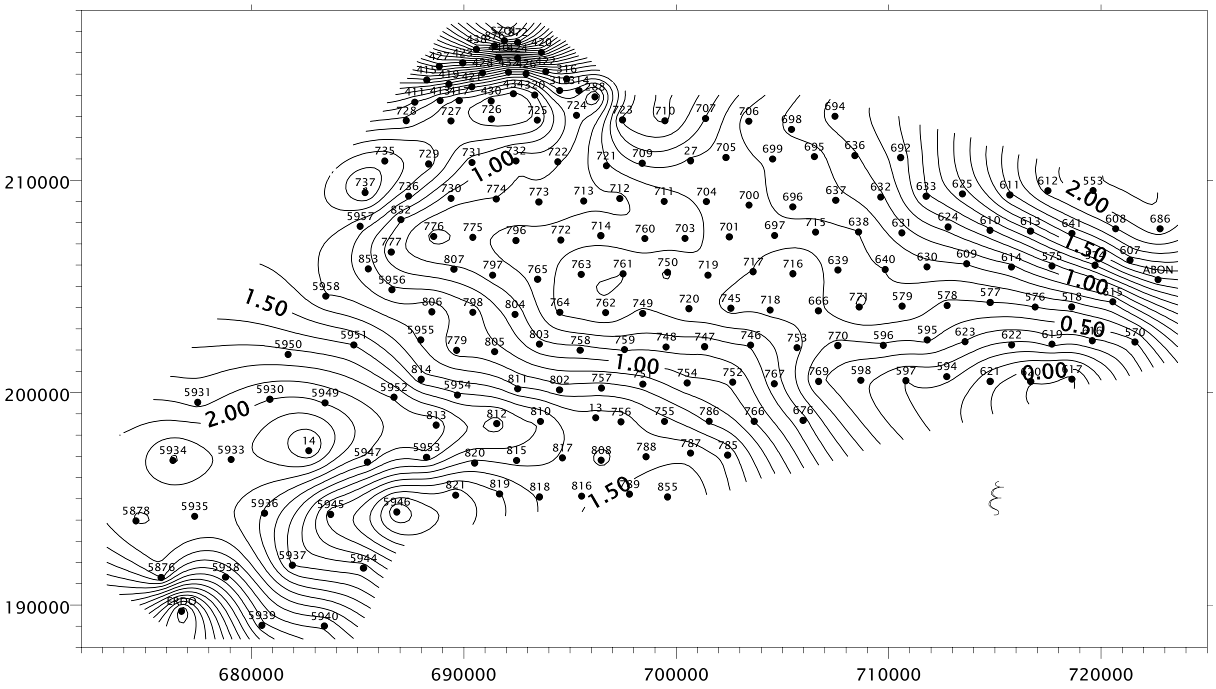

Fig. 5 Interpolated ξ components of deflections of the vertical

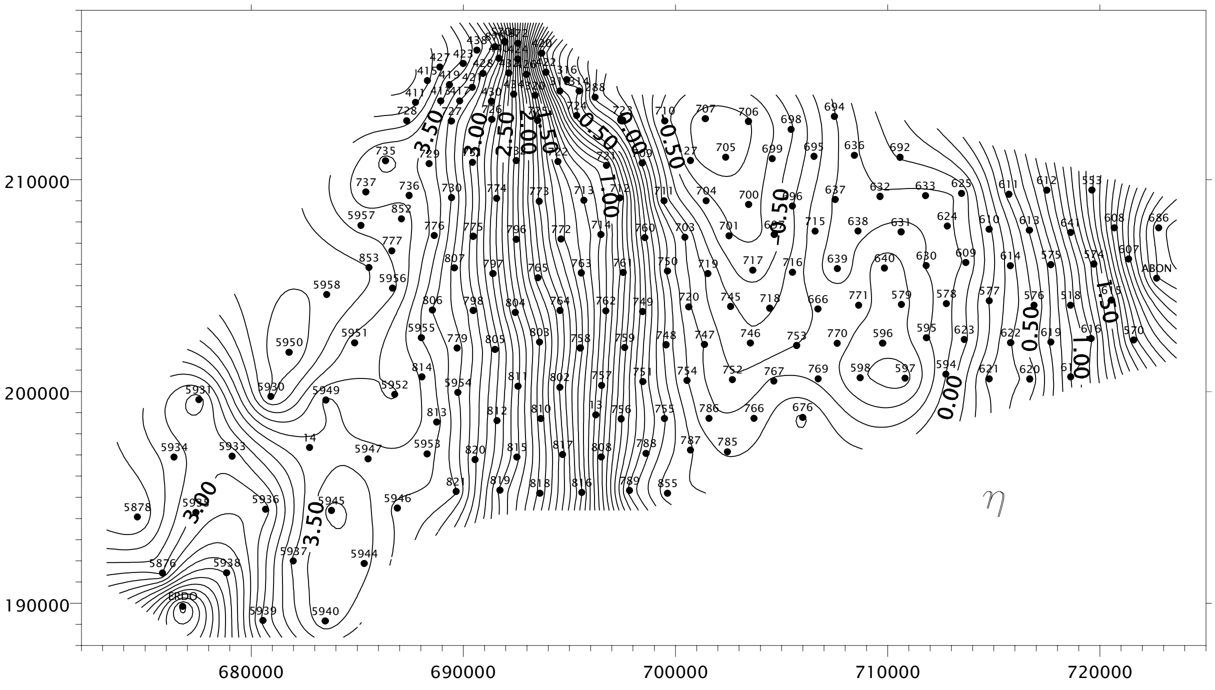

Fig. 6 Interpolated η components of deflections of the vertical

The other possible application of torsion balance measurements is the determination of "fine structure" of local geoid forms. It is possible to compute geoid heights on the torsion balance stations directly, using a new practical solution of astronomical leveling:

$$N_k - N_i = \left(\frac{\xi_i + \xi_k}{2} \cos \alpha_{ik} + \frac{\eta_i + \eta_k}{2} \sin \alpha_{ik} \right) s_{ik} \qquad (3)$$

where s_{ik} is the distance between points i and k and α_{ik} is the azimuth between the two points (VÖLGYESI, 2001). Based on the previously interpolated deflection of the vertical components, geoid computations were carried out. The computed geoid map can be seen on *Fig. 7.* Standard deviation of geoid height differences at checkpoints, computed directly over the arbitrary-shaped network of torsion balance stations is ±0.04m (VÖLGYESI, 2001). This value of standard deviations of computed geoid heights confirm the fact that torsion balance measurements can be used very effectively for determination of fine structure of local geoid forms.

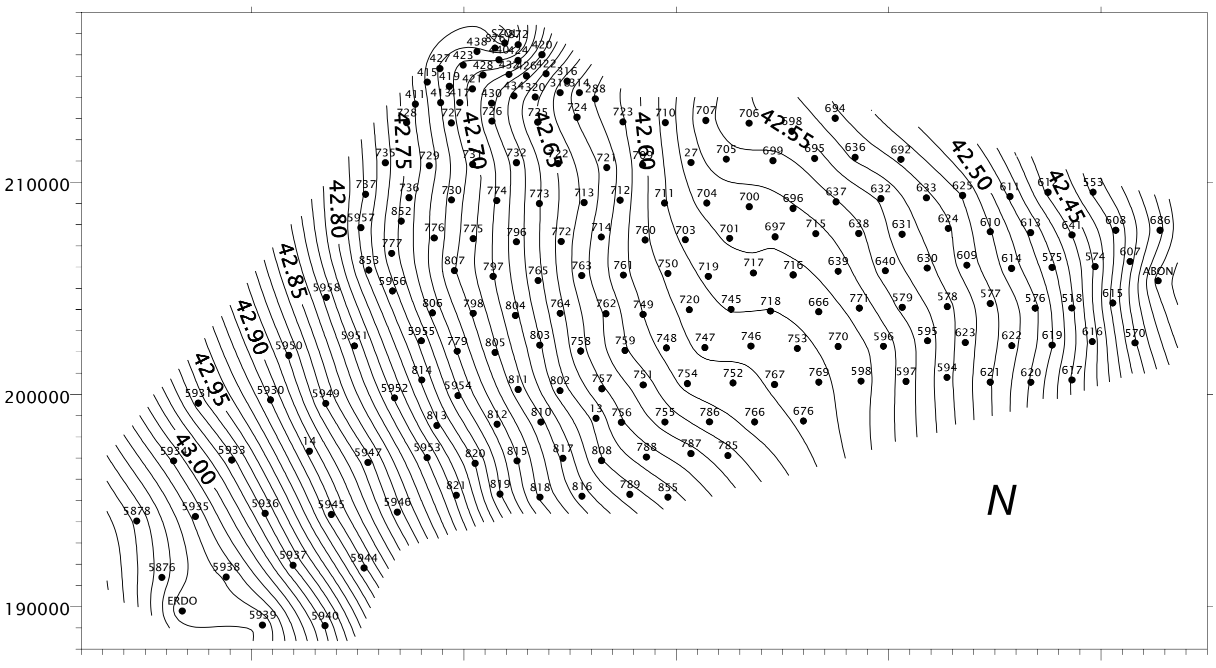

Fig. 7 The computed geoid map

Another possible application of torsion balance measurements is the prediction of gravity anomalies by least squares collocation (TÓTH, 2000). Knowledge of gravity anomalies is very important for numerous geodetic applications - mainly for geoid determination. Results of the test computations demonstrated that torsion balance measurements can advantageously applied for gravity field determination in Hungary. Accuracy of predicted gravity anomalies is ±1 mGal (TÓTH, 2000).

A new fine possibility comes from the theoretical results of van Gelderen and Rummel (VAN GELDEREN, RUMMEL 2000). They give a theoretical solution of surface

integrals of certain combinations of elements of Eötvös tensor, giving a possibility of application of combinations of horizontal gradient W_{zx} , W_{zy} and the curvature data W_Δ , $2W_{xy}$ for the solution of geodetic boundary values problem. In the near future we plan to try a practical application of this new method.

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